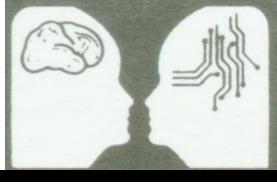


Lecture 13.2 Models of Topography and Map Patterns

Reading Assignments

From the Textbook

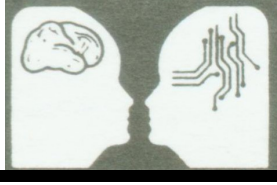
Section 13.4

Suggestions for Further Reading

Retinotopy: Willshaw & von der Malsburg (1979), Hjorth et al. (2014)

SOM model: Kohonen (1982), Obermayer et al. (1990)

Q1: How are topographic maps established?



First we focus on the largest-scale structure: the topographic mapping between neural regions.

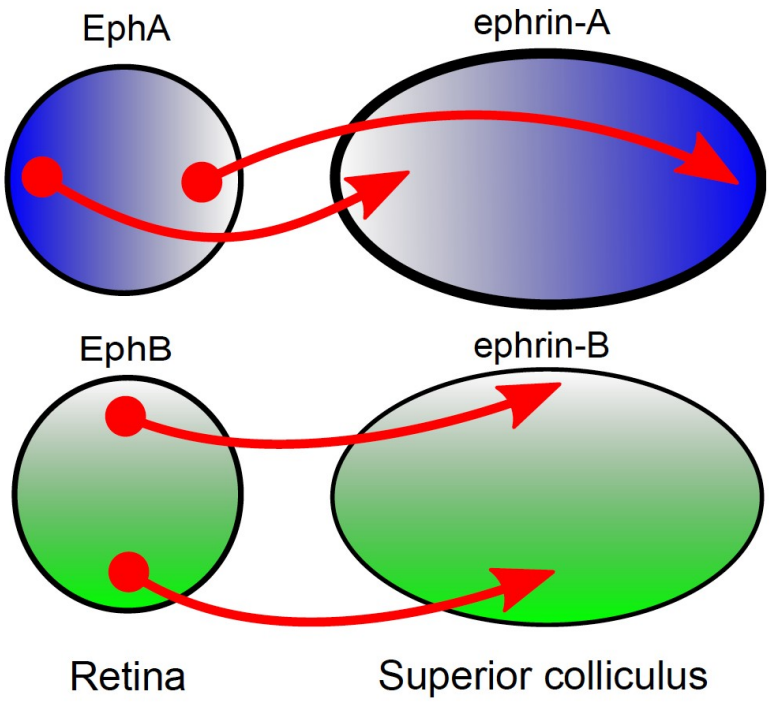
Concrete example: How do the retinal ganglion cells in the eye connect to their targets in the superior colliculus in mammals (or optic tectum in other species)?

Possible mechanisms (reviewed in Hjorth et al. 2014):

- **Chemoaffinity:** Genetic labels on source specify matching targets
- **Neural activity:** Local correlations on retina give clues to topography
- **Competition between incoming axons:** To establish specificity

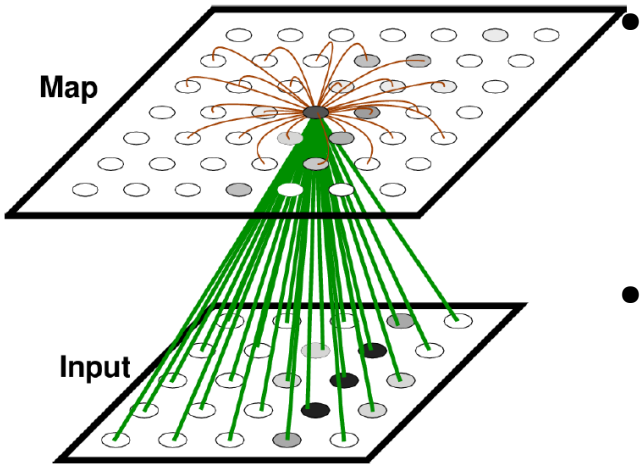
We first describe the chemoaffinity models briefly, and then consider neural activity models in more detail because they share mechanisms with the models considered in subsequent sections.

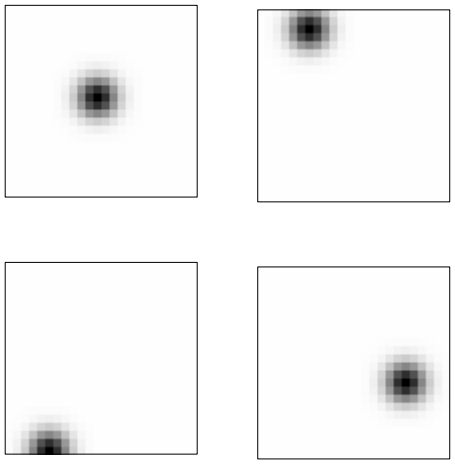
Retinotectal mapping via chemoaffinity



- Gradients of chemicals called Ephs provide a 2D retinal coordinate system.
- Axons from the eye to the superior colliculus sort out their relative locations based on corresponding gradients of ephrins at the target.
- This basic process does not appear to require neural activity but can be affected by it.
- Most models in the other lectures for chapter 13 start from an assumption that this chemical guidance process has completed successfully.
- For the first model we consider in detail, we examine whether activity alone would have been sufficient (even though it is clearly only part of the explanation for the actual retinotectal mapping).

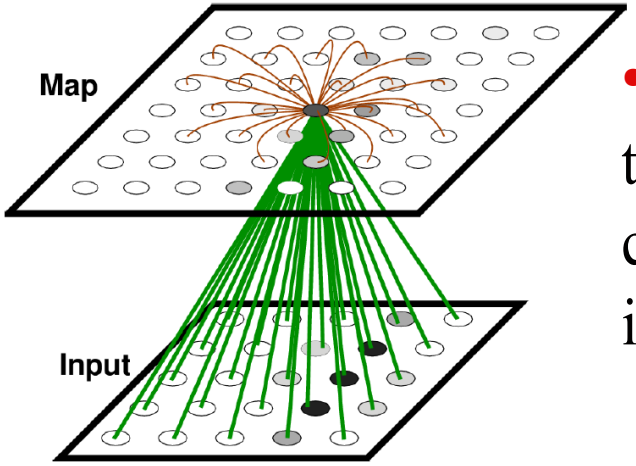
SOM model of neural maps





- Kohonen (1982) self-organizing map (SOM): 2D sheet of "neurons" with weighted connections from an input vector (here a 2D matrix).
- Focuses on how the 2D map organization comes to reflect properties of the input patterns.
- Highly abstracted from neural processing, and includes only activity-dependent mechanisms.
- First we consider a very simple model of initial formation of topographic maps, such as a retinotectal map.
- Input patterns will be random localized patches of activity, as in spontaneous retinal waves from early development, varying only in (x, y) location.

SOM model activation



- For a given input pattern, an initial measure of the response v of unit (k, l) in the map is computed as a Euclidean distance between the input vector $\boldsymbol{s}$ and the weight vector $\boldsymbol{w}_{kl}$:

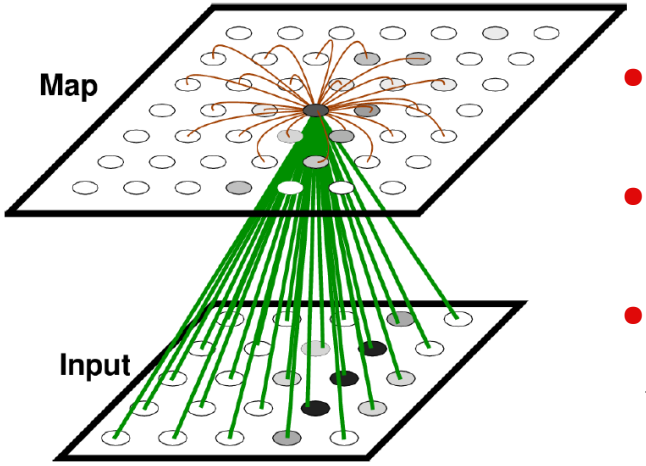
$$v_{kl} = \|\boldsymbol{s} - \boldsymbol{w}_{kl}\|$$

- A "winning neuron" is then selected, unit (r, s) with smallest v_{rs}
- To model lateral interactions, the activity $h_{rs,ij}$ for every unit (i, j) is then computed as a function of the horizontal distance from the winning neuron:

$$h_{rs,ij} = \exp\left(-\frac{(r-i)^2 + (s-j)^2}{\sigma_h^2}\right)$$

- Here the neighborhood function is a 2D Gaussian with radius σ_h

SOM model learning

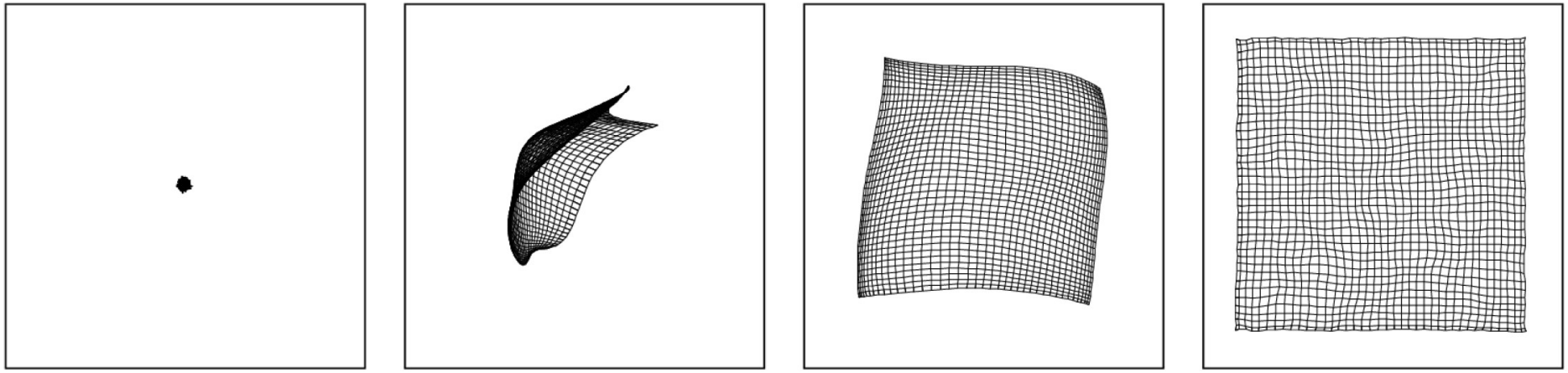


- The weights w_{kl} are initially random.
- Thus the initial "winner" is also random.
- The weight $w_{k,ij}$ from each input unit k to unit (i, j) is adjusted by a Hebbian-like rule:

$$w'_{k,ij} = w_{k,ij} + \alpha(s_k - w_{k,ij})h_{rs,ij}$$

- The learning rate α determines how much the weights change.
- Both α and the neighborhood radius σ_h are reduced gradually.
- Initially all weight vectors make a big jump toward each input vector.
- Eventually only a single neuron and its immediate neighbors move, and only slightly, to allow the weights to settle into stable values.

SOM results: Unfolding to represent a 2D space



(a) Iteration 0: Initial (b) 1000: Unfolding (c) 5000: Expanding (d) 40,000: Final

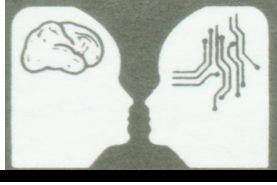
(a) Center-of-gravity grid plots: Initially, weights to all neurons average to the center of the retina (since all weights are random over the full retina).

(b) Neurons gradually differentiate through learning of random example inputs, while remaining similar to neighbors.

(c) As neighborhood size decreases, neurons can become more distinct.

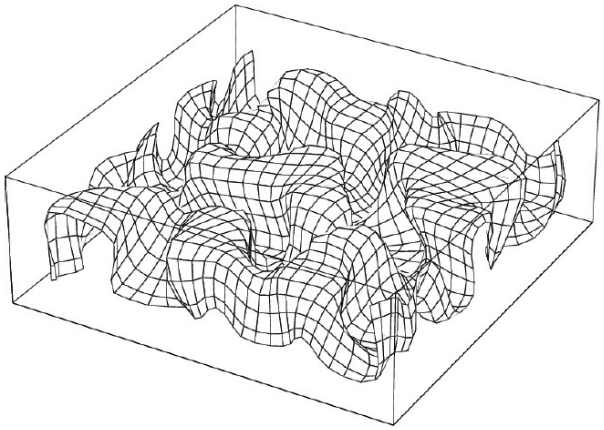
(d) Eventually neurons develop preferences distributed throughout the input space; connecting each preferred location forms a retinotopic grid.

Q2: Why do feature maps have their observed patterns?

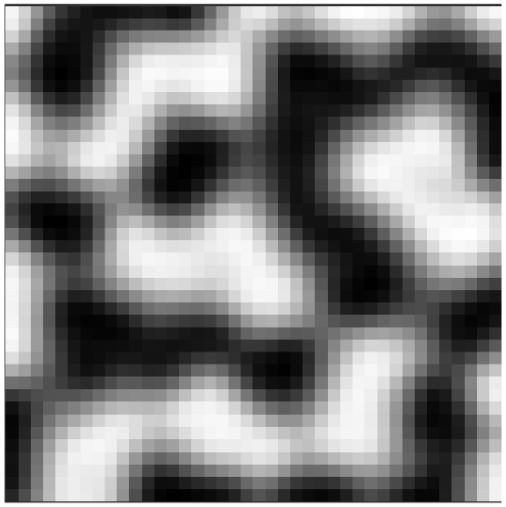


- Given inputs that vary in 2D (i.e., *x,y* location), the SOM thus formed a map of 2D position across the cortical surface.
- The actual input to the cortex varies in many more dimensions than 2, considering all the possible shapes of spontaneous activity patterns, as well as all the possible images experienced postnatally.
- What happens when the number of dimensions of variance in the input is increased, e.g., by adding an additional eye and choosing an input in only one eye or the other?
- Input is now chosen from a 3D space (x, y, e) instead of the 2D space (x, y).
- How will the map patterns change?

SOM feature maps of multiple dimensions



SOM map of the 3D (x, y, e) space



Visualization of ocular dominance

- For inputs distributed in the 3D space (x, y, e) the 2D SOM map "folds" in the third dimension.
- Each local area of the 2D map has a preference for both location and eye, and the eye preference alternates across the 2D surface.
- When the eye preference is plotted on the cortical surface rather than in the input space, the plot resembles ocular dominance patterns.
- Perhaps feature map patterns arise as a network learns to cover a multidimensional input space using a 2D cortical space?
- Similar SOM models of other dimensions, e.g., orientation, also give realistic map patterns.