Lecture 3.3 Exercises:

(adapted from the book by J. Keener and J. Sneyd, Mathematical Physiology, 1998 edition) A central pattern generator (CPG) is a neural circuit that can generate its own rhythmic activity without rhythmic input (see remarks at the ends of Section 3.1.3.2 and Section 3.3.3). Consider a model CPG consisting of two oscillators coupled with depressing synaptic inhibition, given by

$$v_i' = (f(v_i, w_i) - s_i g_s(v_i - v_{syn}) + i_{app}$$

$$\tau w_i' = w_\infty(v) - w_i$$

$$s_i' = \alpha_s(1 - s_i)x_j F(v_j) - \beta_s s_i$$

$$x_i' = \alpha_x(1 - x_i) - \beta_x F(v_i) x_i$$

where $i = 1, 2$, $j = 3 - i$, $f(v, w) = 1.35v(1 - v^2/3) - w$, $w_\infty(v) = \tanh(5v)$, $F(v) =$

$(1 + \tanh(10v))/2$. The idea of this model is that if v stays high, then x decays and limits the growth of s.

1. Simulate the model with parameters $\tau_v = 5$, $v_{syn} = -2$, $\alpha_s = 0.0025$, $\beta_s = 0.002$, $\alpha_x = 0.001$, β x = 0.01, $g_s = 0.19$, $i_{app} = 0$ (e.g., with the file ks.m). You should see a nice anti-phase rhythm.

2. Draw phase portraits (e.g., with ksnull.m) to explain how the oscillation works. Explain why, with the given parameters, this is not in fact a good representation of a CPG. (Hint: What happens with different initial conditions? What effects is the inhibition having?)

3. Explore different initial conditions with larger g_s, so that inhibition has a larger effect. In particular, try to make the model exhibit transitions by escape and by release, for two different sets of parameters. In each regime, check what happens as i app is varied and assess the role of the synaptic depression term *x*.