Lecture 3.1 Exercises

3.1.1 Spinal cord

For these, for equations (3.2) with the "steady state" functions expressed as

$$x_\infty = \frac{1}{\left[1 + \exp\left(\frac{u - \theta_x}{k_x}\right)\right]}$$

for $x \in \{a, d, s\}$, the relevant parameter values are: $\theta_a = 0.18$, $k_a = -0.05$, $\theta_d = 0.5$, $k_d = 0.2$, $\theta_s = 0.14$, $k_s = 0.02$, while the time constants are $\tau_a = 1$, $\tau_d = 2$, $\tau_s = 500$ (see Table 1 of Tabak et al., J. Neurosci., 2000).

1. Develop a Matlab code for the 3-variable model of episodic rhythm in developing spinal cord. Integrate numerically (say by the Euler method) the differential equations (equations 3.2 in chapter 3) to generate the rhythm and plot the time courses of *a*, *d* and *s* (see Fig. 3.4B in chapter 3). Notice, the initial transient phase will depend on your choice of initial conditions. Show that you can choose initial conditions to have the transient phase begin within an episode or within a quiescent state.

2. Vary the time scale of the slow depression variable, *s*, say by doubling the value of τ_s, and doubling again. Show that the rhythm's period scales approximately with τ_s. Why? Does this scaling persist as you decrease τ_s?

3. Fix the value of *s* at 0.9 and reproduce the time course of cycling as in Fig. 3.4A. Explore and describe the behavior as depression, *d*, speeds up by decreasing the value of τ_d, say by 50% and again and again...

4. Suppose that *d* is very fast. Implement the simplification $d = d_\infty(a)$ and reproduce the time courses and phase plane (with nullclines and trajectory) as in Fig. 3.9. Notice that the range of *s* over the rhythm's limit cycle and the period are larger compared with that found in Fig. 3.4B (part 1, above). Explain why. Predict (say, from the phase plane portrait) and then confirm by simulation the effects on episode durations, quiescent phase durations, and period due to changes in the parameter v_s over the range of values (0,1); use at least 10 values. Plot and explain the results.

5. Model the effect of partially blocking recurrent excitation. Re-introduce the synaptic strength parameter, n, as in equation 3.1, into equation 3.2. Consider as the control case $n = 1.2$, and then, at $t= 750$, reduce n to 0.9. Plot the time course of a and s and the trajectory in the $s - a$ phase plane projection. Explain how blocking (reducing n) changes the operating range for s and the longer quiescent state duration.

3.1.2 Binocular rivalry

In binocular rivalry experiments, each eye is shown a different image, yet only one unified image is perceived. A firing rate model for binocular rivalry is given by

$$u_1' = -u_1 + F(I - wu_2 - gz_1)$$

$$z_1' = \frac{u_1 - z_1}{\tau}$$

$$u_2' = -u_2 + F(I - wu_1 - gz_2)$$

$$z_2' = \frac{u_2 - z_2}{\tau}$$

where u_i represents the intensity of the neural representation of the image shown to eye i and the firing rate function F is bounded and monotonically increasing (e.g. Curtu et al., SIADS, 2008). This model is similar to equations (3.3) but with linear dynamics in the equations for the adaptation variables z_i.

1. For $w = 5$, $g = 1$, $\tau = 20$ and $F(x) = 1/(1 + \exp(-(x - 2)))$, compute the bifurcation diagram for the model with bifurcation parameter I. That is, simulate the model with binriv.m for an increasing sequence of values of I and see how the dynamics changes (compare plots of variables versus time, as well as plots in the (u_1, z_1) or (u_2, z_2) plane when needed). Now set $g = 0.25$, repeat this process, and indicate what features have changed.

2. Use simulations with various combinations of (I, g) to compute an approximate two-parameter bifurcation diagram, with respect to parameters g and I, including a curve of bifurcation points of each type.

3. Perform a stability analysis of the homogeneous steady state (i.e., the critical point with $u_1 = u_2$ and $z_1 = z_2$) including finding eigenvalues and eigenvectors for the linearization of the model about this state.

4. Analytically find a condition that yields an Andronov-Hopf bifurcation (and thus oscillations) as the input *I* increases and a condition that yields a non-oscillatory bifurcation (that will give winner-take-all behavior) as *I* increases. Check how this compares to the numerical results.