Lecture 3.2 Exercises:

For each of the following, state whether or not it could actually arise in practice. If not, explain why. If so, provide an example system with this feature:

1. a one-dimensional ODE with an oscillatory solution;

2. a one-dimensional ODE with exactly one critical point p such that $x(t) \to p$ as $t \to \infty$ if $x(t) < p$ but $x(t) \to \infty$ as $t \to \infty$ if $x(t) > p$;

3. a two-dimensional (planar) ODE with two saddle points and no other critical points;

4. a two-dimensional ODE with two nodes;

5. a two-dimensional ODE with a stable node, a stable periodic orbit, and no other stable solutions;

6. a two-dimensional ODE with three saddle points and no other critical points;

7. a two-dimensional ODE with two spiral points.

Hint: It is helpful to note that in a planar system, every periodic orbit (or complete circuit formed of orbits connecting critical points) must have at least one critical point inside (a fact that is actually surprisingly hard to prove).