Lecture 2.5 Exercises: Network models

5.1 A sparsely connected associative memory model

Consider the Tsodyks-Feigelman model with binary neurons and parallel synchronous dynamics in discrete time,

$$S_i(t+1) = \Theta\left(\sum_{j=1}^{N} J_{ij} S_j(t) - T\right) \qquad (1)$$

where T is a threshold, and the synaptic matrix is

$$J_{ij} = \frac{c_{ij}}{cf(1-f)N} \sum_{\mu=1}^{p} (\xi_i^\mu - f)(\xi_j^\mu - f) \qquad (2)$$

where c_{ij} is a random connectivity matrix: for each pair (i, j), c_{ij} = 1 with probability c, c_{ij} = 0 with probability 1 − c, and ξ_i^μ are p binary memories, for all i = 1, . . . , N , p = 1, . . . , p, ξ_i^μ = 1 with probability f , ξ_i^μ = 0 with probability 1 − f

1. Suppose that at time t = 0 the network is correlated with pattern μ = 1 but not with other patterns. Specifically, we assume that $S_i(t=0)$ = 1 with probability $M_+(0)$ if ξ_i^1 = 1, while $S_i(t = 0)$ = 1 with probability $M_0(0)$ if ξ_i^1 = 1, and $M_+(0) > M_0(0)$. Show that the synaptic inputs are distributed according to a Gaussian. Compute the mean and variance of synaptic inputs, both for neurons with ξ_i^1 = 1 (foreground neurons) and neurons with ξ_i^1 = 0 (background neurons).

2. When the connectivity is sparse enough, one can show that the 'synaptic inputs' of two distinct neurons are uncorrelated. Assuming this, what are $M_+(t = 1) \equiv \mathrm{Prob}(S_i(t = 1) = 1 | \xi_i^1 = 1)$, $M_0(t = 1) \equiv \mathrm{Prob}(S_i(t = 1) = 1 | \xi_i^1 = 0)$, as a function of $M_+(0)$ and $M_0(0)$ (Hint: use the function $H(x) = \int_x^{+\infty} \frac{dz}{2\pi} \exp\left(-\frac{z^2}{2}\right)$, and define $\alpha = p/(cN)$)?

3. Again when connectivity is sparse enough, one can show that the equations for M_+ and M_0 can be iterated. What are the equations for M_+ and M_0 in the long time limit (i.e. when the network reaches a fixed point attractor)?

4. What is the capacity of the network (maximal value of α for which in the long time limit $M_+ > M_0$), for f = 1/2 and T = 0 (Hint: reduce the system of two equations to a single equation for the overlap $m = M_+ - M_0$, and find the largest α for which the equation has a non-zero solution)?