Lecture 2.3 Exercises: Single neuron models

3.1 Hodgkin-Huxley and Krinsky-Kokoz reduction

- Simulate the Hodgkin-Huxley (HH) model for 500ms for several representative values of the applied current (i.e. I = 0, 10, 20, 50). Plot the dynamics of the membrane potential and the gating variables as a function of time for all these applied currents.
- Modify the code to implement the Krinsky-Kokoz reduction. The Krinsky-Kokoz reduction consists in setting $m=m_\infty(V)$, and $h=1-n$ in the equation for the voltage in the HH model. The equation for the n variable is unchanged. Thus, the model has only two variables (V and n), as opposed to the four variables of the HH model. Simulate the Krinsky-Kokoz reduced model for the same values of the current, plot both variables as a function of time and compare the dynamics with the HH model.

3.2 Quadratic Integrate-and-Fire model

The Quadratic Integrate-and-Fire (QIF) model is described by

$$\tau \frac{dV}{dt} = \frac{1}{V_0}(V^2 - V_0^2) + RI \tag{3}$$

where $V_0 > 0$ is a constant potential.

1. For $I = 0$, draw the flow of V (i.e. put small arrows) on the real line. What is the neuron stable resting state when $I = 0$?

2. Show that for I larger than a critical $I_c = V_0 / R$, there is no longer any stable resting state. What does the membrane potential do in this case?

3. In this model, one assumes a simple spike and reset mechanism: when the neuron potential reaches $V = +\infty$, it is instantaneously reset to $V = -\infty$. Compute the f-I curve $r(I)$. How does $r(I)$ behave for I close to I_c? Compare this behavior with the behavior of the leaky integrate-and-fire neuron close to threshold.

3.3 Integrate-and-fire neuron with firing rate adaptation

We consider a two-variable spiking neuron model:

$$\tau_m \frac{dV}{dt} = -V - W + I_{syn}(t) \quad (4)$$

$$\tau_w \frac{dW}{dt} = -W \quad (5)$$

where V is the voltage, τ_m is the membrane time constant, W is an 'adaptation' variable whose dynamics is governed by a time constant τ_w , and $I_{syn}(t)$ is the synaptic input. In addition, when the voltage reaches the voltage threshold $V_T > 0$ the voltage is reset to $V_R < V_T$ and W is increased by $W_R \geq 0$.

1. Describe qualitatively what happens when a constant suprathreshold input is injected to the model, $I_{syn}(t) = I > V_T$? How does the model differ from the standard leaky integrate-and-fire neuron?

2. We start by making a simple approximation: we assume that W is constant in between spikes. However, W jumps by a value W_R at each spike. The neuron starts with parameters $V = 0$, $W = 0$, $I_{syn}(t < 0) = 0$. At time $t = 0$ the synaptic current is increased instantaneously to a current $I > V_T$ larger than the threshold current for spike generation.
    a. Discuss qualitatively what happens after the first spike.

    b. At which value of W does the model stop spiking?

    c. We suppose that W_R is much smaller than $I - V_T$. Show that the total number of spikes emitted is roughly $(I - V_T)/W_R$.

    d. Compute the duration of an interspike interval as a function of W in that interval.

3. The approximation made above (W constant in between spikes) is too simple and results in a finite number of spikes emitted for any injected current. In order to compute the mean firing rate $r(I)$ in response to an injected current I, after the system has adapted to its equilibrium rate, we have to take Eq. (5) into account. To simplify the calculation, we consider the neuron after it has adapted to its equilibrium rate. Hence, we consider a periodic spike train.
    a. Compute the time course of W between two successive spikes, assuming that immediately after the first of the two spikes $W(t = 0) = W_0$

b. Assume that the periodic spike emission has period T. Show that W_0 is given by

$$W_0 = \frac{W_R}{1 - e^{-\frac{T}{\tau_w}}} \tag{6}$$

c. Making the approximation that W is constant and equal to its average value during the whole interspike interval, show that the period T of spike emission is given by

$$T = \tau_m ln\left(\frac{I - V_R - W_R\tau_w/T}{I - V_T - W_R\tau_w/T}\right) \tag{7}$$

You should assume $T \ll \tau_w$ (in which case the assumption of constant W is indeed legitimate).

d. Show that, as the injected current increases, the neuron firing rate $r(I)$ behaves as

$$r(I) \sim aI \tag{8}$$

with $a = [\tau_w W_R + \tau_m(V_T - V_R)]^{-1}$.