Lecture 2.4 Exercises: Synapse models

4.1 Conductance-based synapses and shunting inhibition

We consider a neuron well below threshold for action potential initiation. The neuron is described by its membrane potential, $V(t)$, that obeys the equation

$$C_m \frac{dV}{dt} = -g_L(V - V_L) - g_E(t)(V - V_E) - g_I(t)(V - V_I) \qquad (1)$$

in which C_m is the membrane capacitance, g_L the leak conductance, V_L the resting membrane potential, $g_E(t)$ the time-dependent excitatory synaptic conductance, V_E the excitatory reversal potential, $g_I(t)$ is the time-dependent inhibitory synaptic conductance, and V_I is the inhibitory reversal potential For the sake of simplicity we set V_L = 0mV, and $V_I = V_L$. We consider here the situation in which the voltage is $V = 0$ at $t = 0$, and investigate the effects of various combinations of excitatory and inhibitory conductances on the voltage response.

1. Rewrite Eq. (1) in terms of the membrane time constant $\tau_m = C_m / g_L$, rescaled conductances $\tilde{g}_E(t) = g_E(t)/g_L$, $\tilde{g}_I(t) = g_I(t)/g_L$,, and V_E.

2. Show that Eq. (1) can be rewritten as

$$\tau_{eff}(t) \frac{dV}{dt} = -V + V_{eff}(t) \qquad (2)$$

where $\tau_{eff}(t)$ and $V_{eff}(t)$ are a function of τ_m, $\tilde{g}_E(t)$, $\tilde{g}_I(t)$, and V_E.

3. Excitatory input alone: we consider a situation in which there is no inhibition. The (rescaled) excitatory conductance opens abruptly at $t = 0$, and closes abruptly at $t = \tau_E$,

$$\tilde{g}_E(t) = \begin{cases} 0 & t < 0 \\ g_E & t \in [0, \tau_E] \\ 0 & t > \tau_E \end{cases} \qquad (3)$$

   a. Compute the response of the voltage (EPSP, excitatory post-synaptic potential) in both intervals $t \in [0, \tau_E]$ and $t > \tau_E$. Sketch qualitatively the shape of the EPSP.

   b. What is the amplitude of the peak of the EPSP? Discuss qualitatively how it depends on g E, V_E and the ratio of time constants τ_E / τ_m.

c. What is the decay time constant of the EPSP?

4. Inhibitory input alone: we consider the reverse situation in which there is no excitation, and the (rescaled) inhibitory conductance opens abruptly at $t = 0$, and closes abruptly at $t = \tau_I$,

$$\tilde{g}_I(t) = \begin{cases} 0 & t < 0 \\ g_I & t \in [0, \tau_I] \\ 0 & t > \tau_I \end{cases} \qquad (4)$$

Compute the response of the voltage (IPSP, excitatory post-synaptic potential). How does it look like?

5. Excitation and inhibition together: we now consider the situation in which there is a tonic inhibitory conductance, $g_I(t) = g_I$. The excitatory conductance again opens abruptly at $t = 0$, and closes abruptly at $t = \tau_E$,

$$\tilde{g}_E(t) = \begin{cases} 0 & t < 0 \\ g_E & t \in [0, \tau_E] \\ 0 & t > \tau_E \end{cases} \qquad (5)$$

a. Repeat the three steps of question (3). At each step, compare what happens with and without inhibition.

b. Do excitatory and inhibitory inputs sum linearly?

4.2 Short-term synaptic depression

We have seen in the course that some synapses undergo activity-dependent reduction of their transmission capabilities, called short-term synaptic depression (STD). STD is well described by the Tsodyks-Markram (TM) model. Here, we consider a simplified version of the TM model, in which we no longer consider the response of the synapse to individual presynaptic spikes, but rather to the presynaptic firing rate In this model, the fraction of available vesicles $x(t)$ obeys the equation

$$\dot{x} = \frac{1 - x}{\tau_D} - Uxv(t) \qquad (6)$$

where $0 < U < 1$ is the release probability, $\tau_D \sim 100$ms is the recovery time constant, and $v(t)$ is the firing rate of the pre-synaptic cell. The post-synaptic current, $I(t)$, is given by

$$I(t) = AUx(t)v(t) \tag{7}$$

where A is a constant.

1. Write down a solution to Eq. (6) for an arbitrary $v(t)$.

2. Suppose the pre-synaptic cell has been firing at a steady rate $v(t) = v_0$ long enough for x to reach a steady state value x_0. Write down x_0 as a function of v_0.

3. We consider now a step increase in presynaptic firing rate. At $t = 0$, $v(t) = v_0$ and $x(t) = x_0$, and the pre-synaptic firing rate instantaneously changes to $v_1 = v_0 + \Delta v$ ($\Delta v > 0$) and then stays constant.
   a. Compute and sketch the post-synaptic current $I(t)$ as a function of time.

   b. Sketch how the step increase in current at $t = 0$, $\Delta I = I(0^+) - I(0^-)$, depends on v_0.

   c. Show that in the limit of large v_0, ΔI is proportional to $\Delta v/v_0$.

   d. Show that in this limit, the steady-state current becomes independent of the pre-synaptic firing rate.

   e. What is the time scale over which the current reaches its new steady value?