

## Lecture 11.2 Exercises

### ***Multi-stage memory systems with $1/\sqrt{t}$ decay***

In section 11.3.3 of the textbook, we derived the formulae for the initial signal to noise ratio of a model with a heterogeneous population of synapses in learning rates are different for different synapses. The synapses were divided into  $m$  different groups of equal size. When the learning rates  $q_k$  were chosen to be equally spaced on a logarithmic scale, we showed that the memory signal to noise ratio decays approximately as  $1/t$  (or, equivalently,  $1/p$ , if one considers the situation in which new memories are stored at a constant rate).  $F(k)$  was the fraction of synapses in subpopulation  $k$  and it was assumed to be constant ( $F(k)=1/m$ ). Now consider the more general case in which  $F(k)$  is arbitrary and it can change with  $k$ .

The exercise is to determine the  $F(k)$ s such that the memory signal to noise ratio decays as  $1/\sqrt{t}$  as in Benna & Fusi (2016). As the decay should be slower, you will have to consider a situation in which  $F(k)$  increases with  $k$ . Derive the formula for the signal to noise ratio as a function of  $t$  and compare it to the case in which  $F(k)$  is constant. In particular consider the memory capacity and the initial signal to noise ratio, as usual.

As you will notice, the initial signal to noise ratio will be tiny compared to the one in the  $1/t$  case. However, you should keep in mind that the situation analyzed in this exercise is one in which the different synaptic populations do not interact. This is one interesting case in which interactions in the form of memory transfer can dramatically change the performance (see Benna & Fusi, 2016).

### ***References***

M.K. Benna and S. Fusi. Computational principles of synaptic memory consolidation. Nature Neuroscience doi:10.1038/nn.4401, 2016.