Lecture 11.1 Exercises

Building synapses with partially unstructured plasticity

The formulae for the memory signal to noise ratio have been derived for the dense case of binary synapses (section 11.3.3 of the textbook). Consider now the sparse case in which the synapses are still binary, but the neuronal activity is sparse. More specifically, assume that the memories are random and uncorrelated, but the neurons are now active with a probability $f \ll 1$. The learning rule is the one described in Amit & Fusi (1994): when both neurons are simultaneously active, the synapse is potentiated with probability q_+. If the pre-synaptic neuron is active and the post-synaptic one is inactive, the synapse is depressed with probability q_-. In the case in which $q_-=q_+f$, the probability of modifying a synapse scales as f^2, greatly increasing the memory capacity. However, consider that the initial signal to noise ratio is reduced by a factor f. In the optimal case, when f scales as $1/N$, the number of memories that can be stored scales as N^2.

Derive the formulae for the initial signal to noise ratio and the memory capacity for the following two models:

1. Hebbian LTP and unstructured depression: the synapses are potentiated with probability q_+ when the two neurons connected by the synapse are simultaneously active. In all the other cases (pre active, post inactive; pre and post inactive; pre inactive and post active), depression occurs with probability z. LTP is structured, as it depends on the pre and post-synaptic activity, whereas depression is unstructured.

2. Structured LTD and unstructured potentiation: the synapses are depressed with probability q_- when the pre and post synaptic neurons are in a different activation state. Otherwise, the synapses are potentiated with probability r.

For both models compute both the initial signal-to-noise ratio and the signal-to-noise ratio after the storage of p random and uncorrelated memories. In particular consider the signal-to-noise ratio of a single neuron that has C synapses on its dendritic tree.

Determine the maximal memory lifetime in both cases and study its scaling properties when r and z are allowed to scale with f. In particular determine what is the best scaling. Which model is more efficient in terms of memory capacity when the initial signal to noise ratio is the same? How do they compare to the scaling properties of the structured model studied in the class?

The role of background noise

Different neural representations (0, 1 or ±1) lead to different scaling properties of the noise. Consider now a representation in which the neurons are either active (with probability f) or inactive (with probability 1-f). Active neurons have activity 1, whereas inactive neurons are noisy, random Gaussian variables with zero mean and variance σ. Determine the scaling properties of the memory signal to noise ratio when the Hebb rule+depression (the one discussed in the class for the sparse case) is used. Determine whether there is a scaling of σ with f that allows us to recover the scaling obtained in the 0,1 case.

References

D. J. Amit and S. Fusi. Dynamic learning in neural networks with material synapses. Neural Comp., 6:957–982, 1994.