Lecture 11.3 Exercises

Working memory, a simple firing-rate model of self-sustained persistent activity

Consider one excitatory population of neurons, with its population firing rate *r* as a function of input current *I* given by:

$$r = F(I) = \frac{aI - b}{1 - \exp(-d(aI - b))}$$

which captures the current-frequency function of a leaky integrate-and-fire neuron, driven by a noisy input. The parameter values are a = 270 Hz/nA, b = 108 Hz, d = 0.154 sec. Because of the recurrent excitation, the current I includes an external input and a recurrent input from the same neural population. The latter is described by a synaptic drive variable s that obeys:

$$\frac{ds}{dt} = F(I)\gamma(1 - s) - s/\tau_s$$

where $\gamma = 0.64$. $I = g_E s + I_{ext}$. The synaptic time constant τ_s = 100 ms. The synaptic coupling strength g_E = 0.28 nA and $I_{ext}=I_0+I_1$ where I_0=0.32 is a baseline input and I_1 represents transient input pulses.

1.
   a. With initial condition s=0, show that the system is at a resting state, and determine the firing rate at that state.

   b. Use a brief (300 ms) positive current of increasing amplitude I_1, when this amplitude is sufficiently large, show that a persistent activity is produced that outlasts the transient input. What is the critical input current amplitude? What is the firing rate of persistent activity? Show that persistent activity can be switched off with a second brief (300 ms) current of negative value.

2.
   a. Study the steady state solutions of the equation. Graphically determine the stable steady states by plotting ds/dt versus s. Do their firing rates agree with the numerically determined firing rates of the resting state and persistent activity state from (1)?

   b. Plot ds/dt versus s for I_1 of three conditions: (1) without the pulse, (2) with a positive pulse above the critical value you found in 1(b), and (3) with the negative pulse used to switch off the persistent activity. Use these plots to explain how external input switches the system between the two stable steady states.

3.

a. Decrease the value of g_E incrementally (by steps of 0.01 nA), and show that persistent activity disappears when the recurrent excitation is below a critical value. What is this critical level of recurrent excitation? Plot ds/dt versus s for g_E = 0.28 nA and for g_E below the critical level, and use it to explain the loss of persistent activity.

b. Increase the value of g_E incrementally (by steps of 0.01 nA), and show that the low-rate resting state is lost when the recurrent excitation is above a critical value. What is this critical level of g_E? Plot ds/dt versus s for g_E = 0.28 nA and for g_E above this critical level, and use it to explain the loss of the low-rate resting state.